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CHARACTERISTICS OF EDGE CUTSET MATRIX OF PETERSON GRAPH WITH ALGEBRAIC GRAPH THEORY

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Abstract: In this work Basic concepts of algebraic graph theory and its properties are reviewed and extended to the related concepts of edge cutset matrix in Peterson graph and its properties. The relation between edge cutset matrix and incidence matrix is Introduced Rank of the Peterson graph edge cutset matrix is also reviewed.

Key Words: Peterson graph – incidence matrix edge cutset matrix – Rank of the Peterson graph.

Introduction

Algebraic graph theory can be viewed as an extension to graph theory. In Which algebraic methods are applied to graph theory problem. Edge cutsets are of great importance in studying properties off communication and transportation networks. The network needs strengthening by means of additional telephone lines. All cut sets of the graph and the one with the smallest number of edges is the most valuable.

This paper deals with Peterson graph and its properties with cut-set matrix and different cut sets in a Peterson graph. Relation between edge cutset matrix with incidence matrix are explained. Rank of the edge cutset matrix in a Peterson graph is dealt with

Definition 1

The degree of a point v_i in a graph G is the number of lines incident with v_i is denoted by deg (v_i)

A point v of degree o is said to be isolated point .

A point v of degree 1 is said to be pendent vertex.

Definition 2

For any graph G we define

$\delta(G) = \text{Min} \{ \deg v / v \in V(G) \}$

$\Delta(G) = \text{Max} \{ \deg v / v \in V(G) \}$

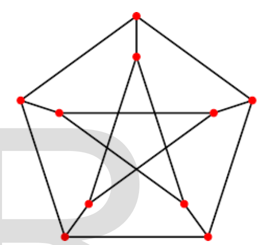
If all the points of G have the same degree then $\delta(G) = \Delta(G) = r$

Then G is said to be a regular graph of degree r

Peterson graph is a regular graph of degree 3

Definition 3

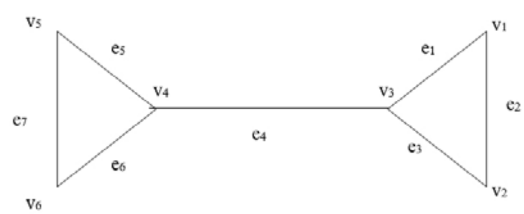
Peterson graph is a 3- regular graph of 10 vertices and 15 edges.



Definition 4

A connected graph G is a edge cutset is a set of edges where removed from G leaves G is disconnected. Provided removal of no proper subset of these edges disconnected G.

A edge cutset always cuts a graph into two



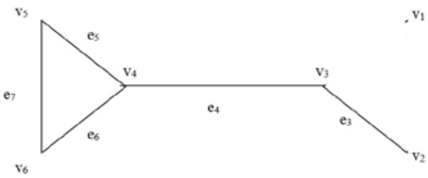
G_1

Edge cutsets of G_1 are

$C_1 = \{ e_4 \}$

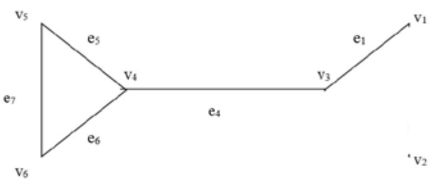
G_1

$C_2 = \{e_1, e_2\}$



G1

$C_3 = \{e_2, e_3\}$



G1

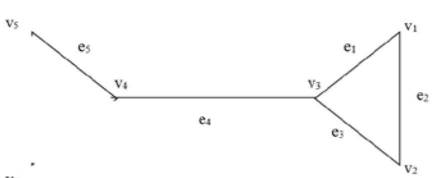
$C_4 = \{e_1, e_3\}$

G1

$C_5 = \{e_5, e_6\}$

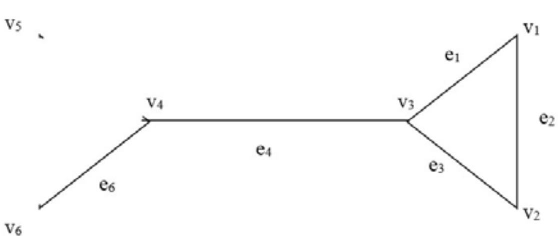
G1

$C_6 = \{e_6, e_7\}$



G1

$C_7 = \{e_5, e_7\}$



Definition 5

Let the graph G have "m" edges and q be the number different cut sets in G.

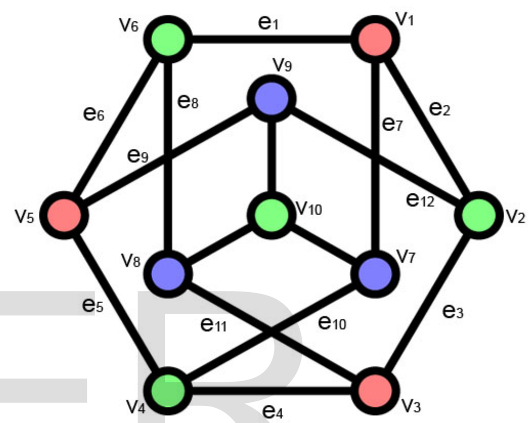
The edge cutset matrix C(G) is given by

$C(G) = (C_{ij})_{q \times m}$

$$C_{ij} = \begin{cases} 1 & \text{If } i^{th} \text{ cutset include } j^{th} \text{ edge} \\ 0 & \text{otherwise} \end{cases}$$

Cut set matrix of the Peterson graph

Cut set matrix of Peterson graph. The different cut sets of the Peterson graph are namely $C_1, C_2, \ldots\ldots\ldots\ldots C_{154}, C_{155}$



$C_1 = \{e_1, e_6, e_8\}$
$C_2 = \{e_1, e_2, e_7\}$
$C_3 = \{e_2, e_3, e_{12}\}$
$C_4 = \{e_3, e_{11}, e_4\}$
$C_5 = \{e_4, e_5, e_{10}\}$
$C_6 = \{e_5, e_6, e_9\}$
$C_7 = \{e_{12}, e_{15}, e_9\}$
$C_8 = \{e_{13}, e_{14}, e_{15}\}$
$C_9 = \{e_8, e_{14}, e_{11}\}$
$C_{10} = \{e_7, e_{13}, e_{10}\}$
$C_{11} = \{e_6, e_8, e_7, e_2\}$
$C_{12} = \{e_1, e_7, e_{12}, e_3\}$
$C_{13} = \{e_2, e_{12}, e_{11}, e_4\}$
$C_{14} = \{e_3, e_{11}, e_{10}, e_5\}$
$C_{15} = \{e_4, e_{10}, e_9, e_6\}$
$C_{16} = \{e_1, e_8, e_9, e_6\}$
$C_{17} = \{e_6, e_5, e_{15}, e_{12}\}$
$C_{18} = \{e_1, e_6, e_{14}, e_{11}\}$
$C_{19} = \{e_1, e_2, e_{13}, e_{10}\}$
$C_{20} = \{e_2, e_3, e_{15}, e_9\}$
$C_{21} = \{e_3, e_4, e_{14}, e_8\}$
$C_{22} = \{e_4, e_5, e_{13}, e_7\}$
$C_{23} = \{e_{12}, e_9, e_{13}, e_{14}\}$
$C_{24} = \{e_{15}, e_{13}, e_8, e_{11}\}$

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$C_{25} = \{e_7, e_{10}, e_{14}, e_{15}\}$
$C_{26} = \{e_6, e_8, e_7, e_{12}, e_3\}$
$C_{27} = \{e_1, e_7, e_{12}, e_{11}, e_4\}$
$C_{28} = \{e_2, e_{12}, e_{11}, e_{10}, e_5\}$
$C_{29} = \{e_3, e_{11}, e_{10}, e_9, e_6\}$
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$C_{152} = \{ e_4, e_5, e_{13}, e_1, e_3, e_{15}, e_9 \}$
$C_{153} = \{ e_4, e_5, e_{13}, e_2, e_6, e_{14}, e_{11} \}$
$C_{154} = \{ e_6, e_5, e_{15}, e_3, e_1, e_{13}, e_{10} \}$
$C_{155} = \{ e_6, e_5, e_{15}, e_2, e_4, e_{14}, e_8 \}$

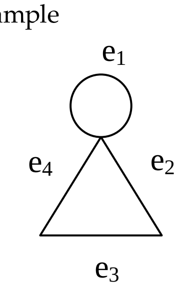
The edge cutset matrix of the Peterson graph. Whose order is 155 x 15 is denoted by C (G).

$$C(G) = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ \vdots \\ C_{155} \end{array} \begin{array}{c} \begin{array}{ccccccccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}_{155 \times 15} \end{array}$$

Results

1) The permutation of rows (or) columns in a edge cutset matrix Corresponds simply to renaming of the edge cutset and edges respectively.
2) Each row in C (G) is a edge cutset vector
3) A column with all zeros corresponds to an edge forming a self loop.

Example



G_1

The graph G_1 has 3 different cut sets Namely

$C_1 = \{e_2, e_3\}$
$C_2 = \{e_3, e_4\}$
$C_3 = \{e_2, e_4\}$

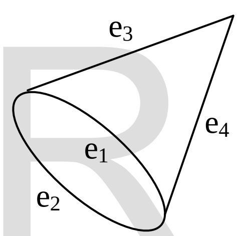
The edge cutset matrix of G_1 is given by

$$C(G_1) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{3 \times 4} \end{array}$$

In a edge cutset matrix all the elements in the first column has zeros and the corresponding edges are self loops. From this it is clear that Peterson graph has no self loop.

4) Parallel edges from identical columns in a edge cutset matrix

Example



Graph G_2

The graph G_2 has 3 different cut sets namely

$C_1 = \{e_1, e_2, e_3\}$
$C_2 = \{e_1, e_2, e_4\}$
$C_3 = \{e_3, e_4\}$

Edge cutset matrix of G_2 is given by

$$C(G_2) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3 \times 4} \end{array}$$

In a edge cutset matrix first two columns are the same. Corresponding edges e_1, e_2 are parallel edges.

v) In a non separable graph since every set of edges incident on a vertex is a edge cutset. Therefore every row of incidence matrix I (G) is included as a row in the edge cutset matrix C (G)

That is for a non separable graph C (G) Contains I (G)

For a separable graph the incidence matrix of a each Block is contained in the edge cutset matrix.

Theorem 1

If G is a connected graph then the rank of a edge cutset matrix C (G) is equal to the rank of the incidence matrix I (G) which is equal to the rank of graph G.

Proof

Let I (G) , B (G) , C (G) be the incidence , Cycle, Cut set matrix of the connected graph G, Then we have

Rank of C (G) $\geq$ n-1 ……………. (1)

Since the number of edges common to a edge cutset and a cycle is always even.

Every row in C is orthogonal to every row in B. Provided the edges in both. Band C are arranged in the same order.

Thus $BC^T = CB^T \cong 0$ (Mod 2)

Rank B + Rank C $\leq$ m

For a connected graph we have

Rank B = m-n+1

$\therefore$ Rank C $\leq$ m – Rank B

Rank C $\leq$ m – (m-n+1)

Rank C $\leq$ n-1

…………. (2)

From (1) & (2)

Rank of C (G) = n-1

Hence the theorem

Result :

If G is a Peterson graph then C (G) is a edge cutset matrix of the peterson graph with 10 vertices and 15 edges

Rank of C (G) = n-1

Rank of C (G) = 9 $\therefore$ By theorem (1)

Conclusion

Peterson graph is a special kind of graph. Cut set matrix is used to communication and transportation network Problems. In addition using the relation. Of incidence matrix, cycle matrix and edge cutset matrix and Peterson graph cut sets matrix rank can be found. Cut set matrix of the Peterson graph has similarity with the regular graph properties of the edge cutset matrix.

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